Too Much Information & The Death of Consensus

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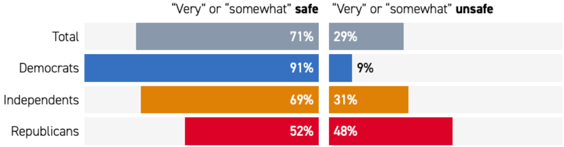
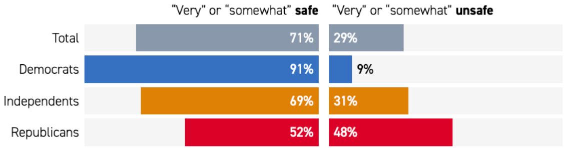
- *Factual* Polarization is less clear:
  - Hypothesis 1 (Post-truth): There has also been factual polarization.
  - Hypothesis 2 (Google): The internet at least produces factual consensus.

Factual Polarisation



Factual Polarisation



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- Iyengar & Peterson 2020: Economic performance indicators, health care policy.

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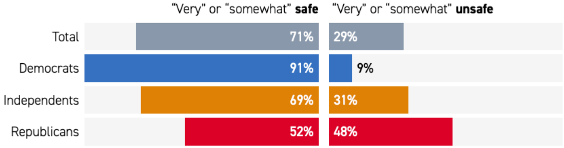

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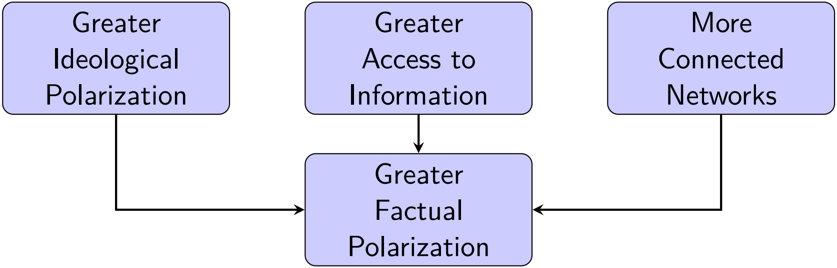
- Iyengar & Peterson 2020: Economic performance indicators, health care policy.
- "*Empirical research on factual belief polarization around citizens' core issue attitudes is surprisingly sparse... Therefore, relatively little is known about factual belief polarization, even (or perhaps especially) at a basic descriptive level.*" Rekker 2022 Some Evidence

Motivated Reasoning

- Thaler 2023, Oprea Yuksel 2022
- Westen et al. 2008 & Moore et al. 2021 present fMRI evidence
- Nyhan 2020 (JEP) "*These tendencies can be especially powerful in contexts like politics [with] strong directional preferences..., low accuracy motives, and lack evidence that would resolve factual disputes.*"
    - Taber & Lodge 2006: Participants presented with contradictory arguments, more likely to counter-argue when disagree.
    - Ditto & Lopez 1992: More likely to question negative medical news

Argument & Preview of Results

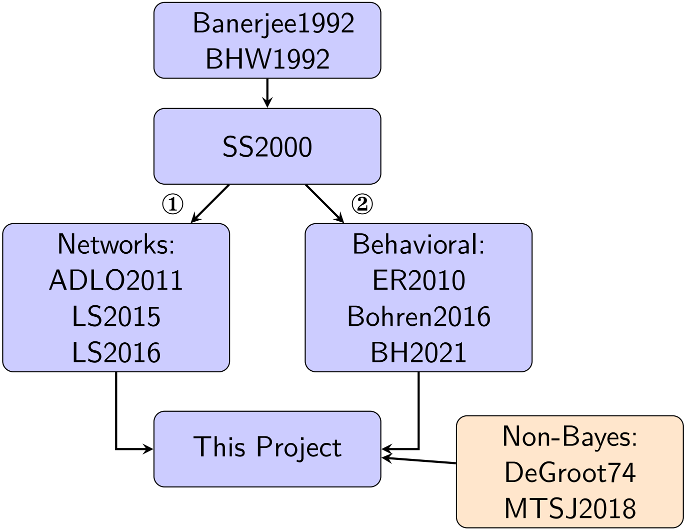
- I take a canonical model of *sequential social learning* and introduce the novelty of *motivated reasoning*.



Argument & Preview of Results II

1. With sufficiently 'precise' private signals, complete learning (i.e. *Bayesians learn*) requires much more connected networks
2. Such private signals also make *consensus* impossible.
   - Consensus can be broken by expanding the support of signals, making ideological polarization more extreme, or both.
3. Polarization can be exacerbated by increasing the clustering and connectivity of a network.
4. In some specific settings, motivated reasoning can help learning.

Literature


Banerjee1992
BHW1992
SS2000
①
②
Networks:
ADLO2011
LS2015
LS2016
Behavioral:
ER2010
Bohren2016
BH2021
This Project
Non-Bayes:
DeGroot74
MTSJ2018

The Model

Model

The basic elements of the model are:

- $\theta \in \{0, 1\}$: $\mathbb{P}(\theta = 0) = \frac{1}{2}$
- $n \in \mathbb{N}$ chooses $x_n \in \{0, 1\}$ to maximize:

$$u_n(x_n, \theta) = \begin{cases} 1 & \text{if } x_n = \theta, \\ 0 & \text{if } x_n \neq \theta, \end{cases}$$

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- Private signal $\varsigma_n \in \mathcal{S} \sim (\mathbb{F}_0, \mathbb{F}_1)$ (mutually a.c., distinct)
- Bayesian *private belief* p_n distributed according to $(\mathbb{G}_0, \mathbb{G}_1)$.
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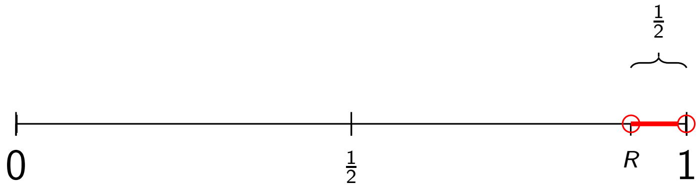
- Type $\tau_n \in \{0, B, 1\}$:
  - $\mathbb{P}(\tau_n = 0) = \mathbb{P}(\tau_n = 1) = \frac{1}{2}(1-\beta)$ i.i.d., $\beta \in (0,1)$
  - Mostly $\beta \approx 0$, though $\beta \approx 1$ highlights fragility of standard model
- The solution concept is (effectively) PBE.

Motivated Reasoning

Consider $\tau_n = 0$:

- $R \in (\frac{1}{2}, 1)$ parametrizes Signal *Rejection*
- $s \in [0, 1)$ parametrizes Prior *Shifting*



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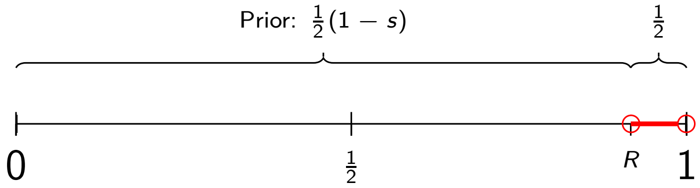

Characterizing Outcomes

- For each agent, let χ_n be the action of their *Bayesian equivalent*.
- Let α_n be their *Bayesian accuracy*: $\alpha_n := \mathbb{P}_\sigma(\chi_n = \theta)$

Definition: Learning & Consensus

Complete Bayesian learning obtains if χ_n converges to θ in probability

$$\lim_{n\to\infty} \alpha_n := \lim_{n\to\infty} \mathbb{P}_\sigma(\chi_n = \theta) = 1$$

Consenus obtains if x_n converges to ω for any $\omega \in \{0, 1\}$ in probability

$$\lim_{n\to\infty} \mathbb{P}_\sigma(x_n = \omega) = 1$$

Stationary Beliefs & Signal Structures

Cascade & Stationary Beliefs I

- Private Beliefs can be *bounded* or *unbounded*:
  - Bernoulli trials (or any finite signal structure) ⇒ bounded.
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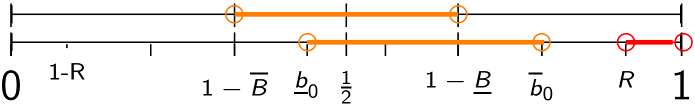


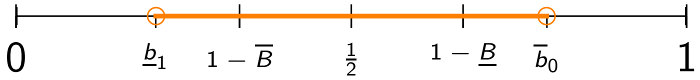
Cascade & Stationary Beliefs II

Definition: Stationary Beliefs

A Bayesian social belief, *sb*, is a *stationary belief* if it is a *cascade belief* for each type.

Nonstationary Signal Structures

- If for $(\mathbb{F}_0, \mathbb{F}_1, s, R)$ there are *no* stationary beliefs, the signal structure is *nonstationary*.
- Otherwise is it *stationary*.



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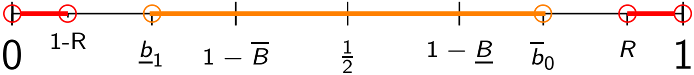

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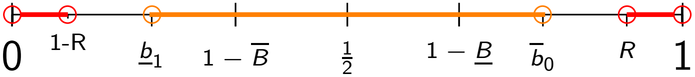
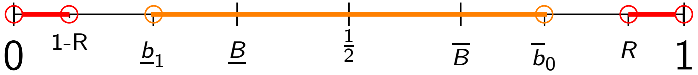
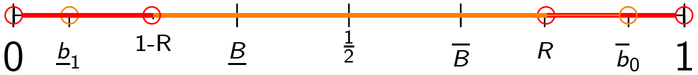


- If $R < \min\{1 - \underline{b}_1, \overline{b}_0\}$, the signal structure is *nonstationary*.
  - Need large s, low R, or large support $[\underline{B}, \overline{B}]$.

Stationary vs Nonstationary Signals



(a) Stationary Signal Structure



(b) Nonstationary Signal Structure

Figure: Signal Structures

Two Agent Example

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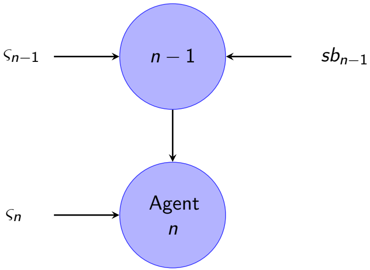
- First consider two Bayesians: Agent n (he) and Neighbor $n-1$ (she)
- Suppose $n-1$ receives a symmetric binary social signal, $sb_{n-1} \in \{0.1, 0.9\}$



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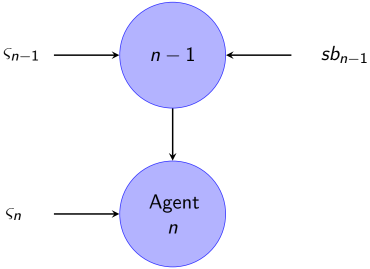
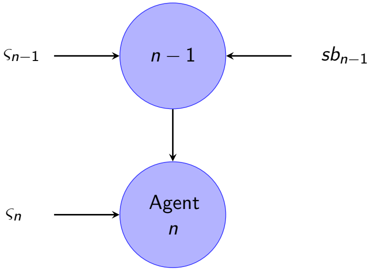


- $\mathbb{P}(x_{n-1} = \theta|\theta) \geq 0.9$ by improvement, and $\mathbb{P}(x_n = \theta|\theta) \geq 0.9$

Two Agent Example II

- But what if $n-1$ is a motivated reasoner?
- Suppose 50/50 type, $s=0$, $R=0.7$



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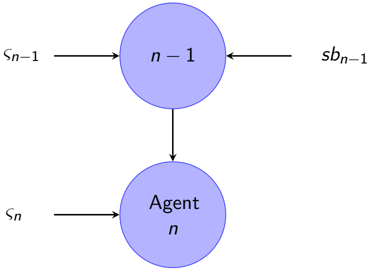

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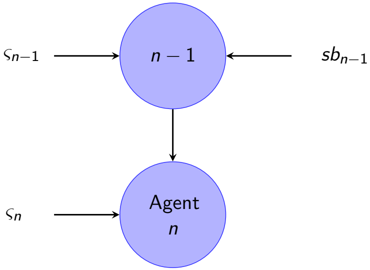
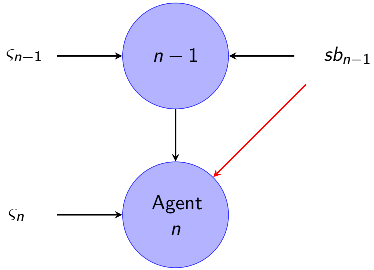


- n observes x_{n-1}, not χ_{n-1}.
- Improves on e.g. $\frac{1}{2}(0.75+0.91)=0.83$ with $\{2(1-\varsigma), 2\varsigma\}$.

Two Agent Example III

- Instead suppose n observes sb_{n-1}



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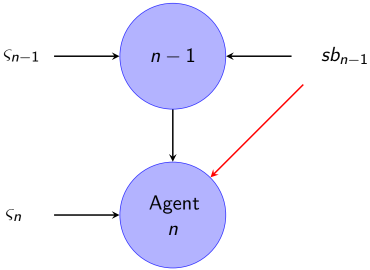

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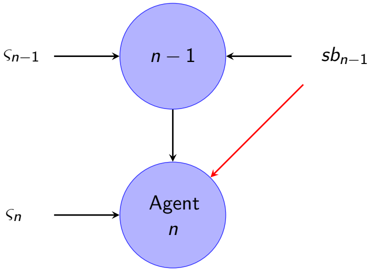

- $n-1$ Bayesian: n improves on sb_{n-1}, observing ς_n and learning about ς_{n-1} through x_n, unless sb_{n-1} is a *Bayesian cascade belief*
- $n-1$ Motivated: n improves on sb_{n-1}, observing ς_n and learning about ς_{n-1} through x_n, unless sb_{n-1} is a *stationary* belief

Learning

Expanding Sample Sizes & Complete Bayesian Learning

Expanding Observations (Deterministic) General Versions

A network topology has expanding observations if we have

$$\lim_{n\to\infty} \max_{b\in B(n)} b = \infty$$

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- We move from *indirect* access to ever more neighbor actions, to *direct* access.

Expanding Sample Sizes & Complete Bayesian Learning

Theorem

Complete Bayesian learning obtains only if the network topology satisfies expanding sample sizes.

Fragility of Correct Bayesian Consensus

- This last theorem can be interpreted as a sort of fragility result, but a more compelling one is the following:

Corollary

In any learning game without ESS where $\beta = 1$ produces correct consensus (a.s. only finitely many Bayesians fail to match the state), setting $\beta < 1$ ensures that a.s. infinitely many Bayesians will instead fail to match the state.

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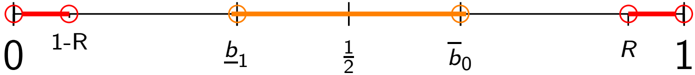
- E.g. from Rosenberg & Vieille 2019 we know that in line networks with very informative private signals we have almost sure learning, but any amount of motivated reasoning breaks this.

Consensus

Consensus

Theorem

1. Complete Bayesian learning implies consensus does not obtain. $(\forall \mathbb{Q})$
2. Thus consensus cannot obtain with nonstationary signal structures. $(\forall \mathbb{Q})$
3. Consensus can occur with stationary signal structures. $(\exists \mathbb{Q})$

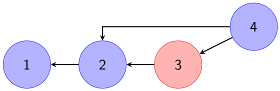


Learning via Motivated Reasoning

Nested Neighbors

Nested Neighbor

m is a *nested neighbor* of n if $\mathbb{Q}(B(m) \subseteq B(n)) = 1$.



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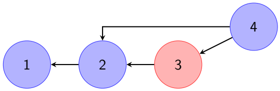

- $B(4) = \{2, 3\}$, 3 is a nested neighbor
- Complete network: all $B(n)$ are nested neighbors.

- If m is a nested neighbor of n, n knows sb_m.
- Each social belief induces an independent binary experiment
  - Defining success parameters $\{p^0(sb), p^1(sb)\}$, Nonstationarity guarantees $p^1(sb) - p^0(sb) > 0 \ \forall sb \in [0, 1]$

Expanding Nested Samples & Learning

Expanding Nested Samples (Deterministic)

For agent n, let $B^n(n) \subseteq B(n)$ be the set of nested neigbors of n. A network topology has expanding nested neighbor samples if for all $K \in \mathbb{N}$, we have:

$$\lim_{n \to \infty} |B^n(n)| = \infty$$

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Theorem 3

If a network topology has expanding nested samples, and the information structure is nonstationary, complete Bayesian learning obtains.

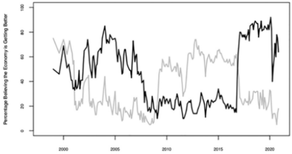
Expanding Nested Samples

- Public commenting, the sequential structure of the game, and the development of more connected & clustered networks lend some credence to this assumption.
- Proposition 2: for any $\epsilon > 0$, there is some $M_\epsilon \in \mathbb{N}$ such that the probability any agent has a rejection-region belief is at least $1 - \epsilon$. Formal Statement
- Other 'Large Sample Principle' results require a core of agents to observe the complete history with some non-zero probability. Royal Family

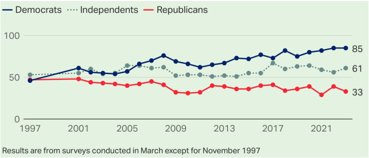
Summary

- ESS Theorem ⇒ Complete Bayesian Learning requires much more connected networks: those with expanding sample sizes. This suggests Bayesian models are fragile.
- Con Theorem ⇒ Consensus is only possible with a stationary signal structure, and our stylized facts give reason to think these consensuses will break. *This can help explain polarization.*
- ENS Theorem ⇒ Ever more clustered & connected neighborhoods reinforce these two results. Expanding nested samples is a sufficient condition for learning with nonstationary signals. Theorem 2 Slides

Factual Polarization II



(a) Economic Perceptions



(b) Global Warming

⏩ Back

Why model Motivated Reasoning this way? Back

- These agents behave *as if* for some $c_1, c_2 \in \mathbb{R}_+$:
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  2. If not, they solve:

$$\arg\min_{msb_n} KL(msb_n, sb_n) + c_2 \times msb_n$$

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- *Social* belief: Oprea Yuksel 2022 & Conlon et al. 2023.
  - Theorems 1 & 3 are robust to applying motivated reasoning to the *whole belief*. Theorem 2 fails.
  - Evidence Recruitment Story: Epley & Gilovitch 2016, Koszegi 2006

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  - Information Avoidance: Oster, Shoulson & Dorsey 2013, Ganguly & Tasoff 2017

Why model Motivated Reasoning this way? ⏩ Back

- These agents behave *as if* for some $c_1, c_2 \in \mathbb{R}_+$:
  1. They can pay c_1 to reject social signals,
  2. If not, they solve:

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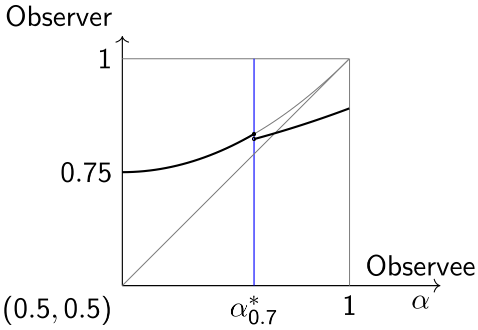
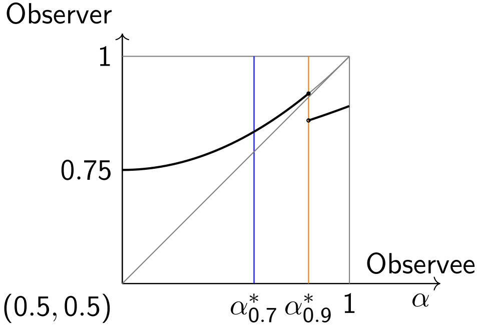
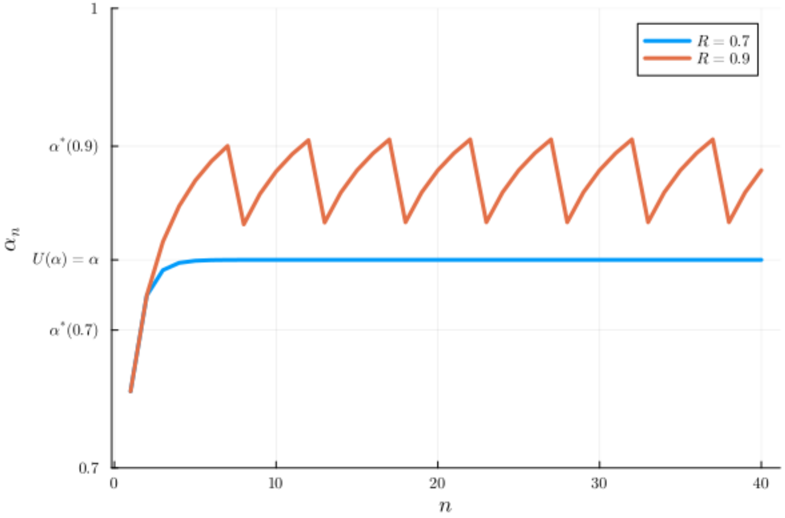
- Can generalize to i.i.d. draw of (R, s), and can smooth out discontinuity without loss.

Line Network Learning



Line Network Learning

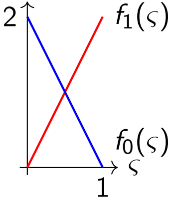



1
$\alpha^*(0.9)$
$U(\alpha) = \alpha$
$\alpha^*(0.7)$
0.7
α_n
0
10
20
30
40
n
$R = 0.7$
$R = 0.9$

Line Network Example Details

Consider a line network with:

- No prior shifting $s = 0$, and $R = 0.7$.
- $\beta = 0$
- $f_1(\varsigma) = 2\varsigma$, $f_0(\varsigma) = 2(1-\varsigma)$ which gives $f_\theta(\cdot) = g_\theta(\cdot)$



$$H(\alpha) := \frac{\alpha}{2}\left[\mathbb{G}_0(\alpha) + 1 - \mathbb{G}_1(1-\alpha)\right] + \frac{(1-\alpha)}{2}\left[\mathbb{G}_0(1-\alpha) + 1 - \mathbb{G}_1(x\alpha)\right]$$

$$\Downarrow$$

$$H(\alpha) = \alpha^2 - \alpha + 1$$

EO & ESS General Versions

Expanding Observations

A network topology has expanding observations if for all $K \in \mathbb{N}$, we have

$$\lim_{n\to\infty} \mathbb{Q}_n(\max_{b\in B(n)} b < K) = 0$$

Expanding Sample Sizes

A network topology has expanding sample sizes if for all $K \in \mathbb{N}$, we have

$$\lim_{n\to\infty} \mathbb{Q}_n(|B(n)| < K) = 0$$

Back

Proposition 2

- Agents have M-nested neighbor samples if they have at least M nested neigbors.

Proposition 2: ENS & Consensus

For any $\epsilon > 0$, there is an $M \in \mathbb{N}$ such that, if agents have M-nested neighbor samples and the information structure is nonstationary, then

$$\liminf_{n\in\mathbb{N}} \mathbb{P}_\sigma(sb_n > R|\theta = 1) \geq 1 - \epsilon$$
$$\liminf_{n\in\mathbb{N}} \mathbb{P}_\sigma(sb_n < 1 - R|\theta = 0) \geq 1 - \epsilon$$

asymptotically agents of non-congenial type each reject their Bayesian social beliefs with probability at least $1 - \epsilon$.

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Theorem 3: Royal Family/Subpopulation Example

- Let $S \subset \mathbb{N}$ be a 'Royal Family', if its members are observed by all following agents.
- In Bala Goyal 1998, the presence of a 'Royal Family' can prevent learning amongst other agents. Here the reverse effect can arise.

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- With $S' = \{3m : m \in \mathbb{N}\} \cup \{1\}$, this is only true for two-thirds of the population.

Can motivated reasoning help with learning?

- Theorem 3 & The Royal Family Example show motivated reasoners helping in some circumstances.
- Beyond this, even without ENS, motivated reasoners can provide a path to learning.

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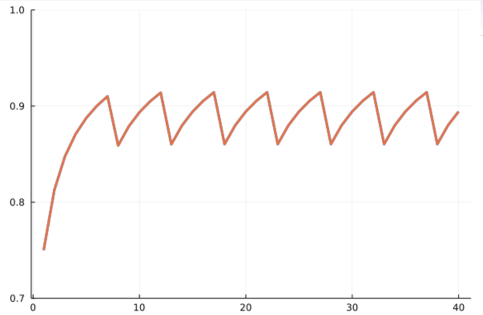
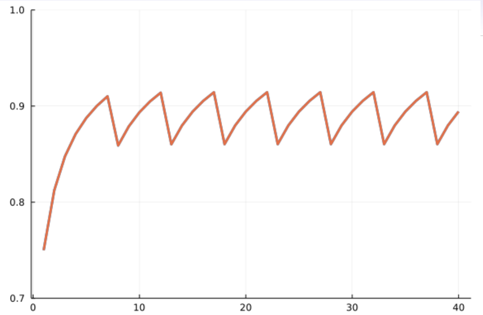
Assume we have:

1. The parameters of our early line network example, with $R = 0.9$
2. $S' = \{10^m : m \in \mathbb{N} \cup \{0\}\}$ forming a line, index these agents by j
3. $S'' = \{j : j \in \{8, 13, 18, 23, ...\}\} \subset S'$
4. Suppose agents in $\mathbb{N} \setminus S'$ satisfy ESS with respect to S''



- Let $Z_j \in \{0, 1\}$ indicate rejection between j and $j + 5$, and $n(j)$ translate j to each agent's actual index.
- From the Law of Total Covariance, we have: $|cov(x_{n(j)}, x_{n(j+5)})| < \frac{1}{2}$

Back



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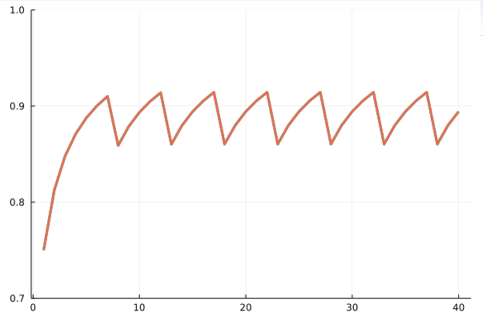

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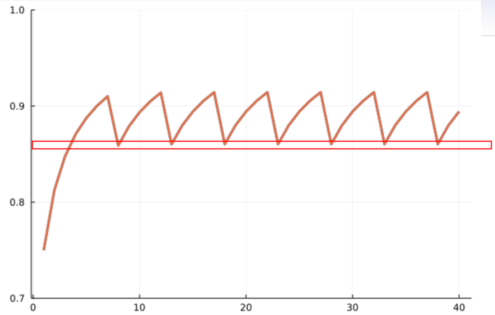
⏩ Back



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