Sleeping Newcomb

The 'Newcomb Tension' in Games with Self-Locating Uncertainty

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Learning with Temporal Ignorance

- I am broadly interested in Bayesian learning, and modelling this in the presence of 'temporal ignorance'.
  - Agents arrive in a game at time t and observe some signal whose distribution depends on this arrival time, but do not perfectly observe it.
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- This temporal ignorance is an instance of *self-locating uncertainty*, as in the *Sleeping Beauty Problem* (wait two slides!)

Self-Locating Uncertainty

- In a game-theoretic setting, one can think of self-locating uncertainty as resulting from two scenarios we normally rule out:

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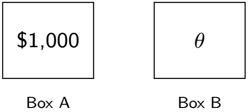
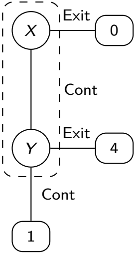
- Today I won't address how (and whether) one should choose to model such games in the first place: look out for *Sleeping Beauty's Dismal Day Out* for this (Yes... I like puns...)
- Instead I discuss a common feature of such games I call a *Newcomb Tension*

Three Puzzles



$$\theta \in \{\$0,\ \$10^{10}\}$$

Newcomb's Problem



AMD

	H	T
$n{=}1,\ d{=}1$	•	•
$n{=}1/n{=}2,\ d{=}2$		•

Sleeping Beauty

Different paradoxes, same structure: locally rational reasoning at the information set disagrees with the planning optimum: a *Newcomb Tension*.

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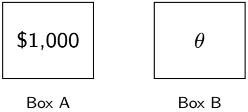

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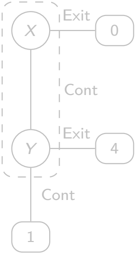

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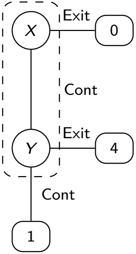

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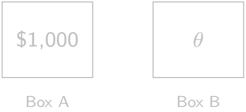

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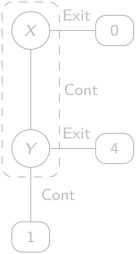

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2. In single-agent games, randomisation always resolves this tension (this is not just a coincidental property of the AMD).
3. In multi-agent games this is not the case (relevant to the duplicating Sleeping Beauty debate).

Model

Single-Information-Set SLU Games

A *single-information-set SLU game* Γ has:

- A finite set of agents N, prior ρ on states Θ, action set $A(h)$.
- A *single* information set $h \subseteq X$, with agent assignment $\iota : h \to N$.
- Co-cardinal vNM utilities $u_n : Z \to \mathbb{R}$.
- No perfect recall: an agent may occupy multiple nodes in h on the same play.
- For each state θ and action profile $\mathbf{a} : h \to A(h)$, let $D(\theta, \mathbf{a})$ denote the *dots* (nodes in h) visited.
- I assume $|D(\theta, \mathbf{a})|$ is deterministic given $(\theta, \mathbf{a})$.

More on the model

Thirder Beliefs

Definition (Thirder beliefs)

Under strategy σ, the interim agent at h assigns

$$\pi(\theta, \mathbf{a}) \;=\; \frac{\rho(\theta)\,\Pr_\sigma(\mathbf{a})|D(\theta, \mathbf{a})|}{\sum_{\theta', \mathbf{a}'} \rho(\theta')\,\Pr_\sigma(\mathbf{a}')|D(\theta', \mathbf{a}')|},$$

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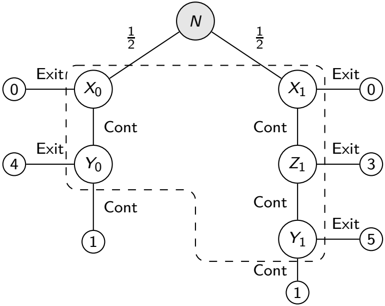
and distributes probability uniformly over dots within each $(\theta, \mathbf{a})$.

- Awakening counts scale the prior; follows Ross's Generalised Thirder Principle, except assuming an equilibrium.
- When $|D(\theta, \mathbf{a})|$ is constant in $(\theta, \mathbf{a})$, thirder and halfer beliefs coincide.

Halfer beliefs

Halfer vs Thirder: Sleeping Beauty Behind the Wheel



AMD calculation

	$\theta=0$			$\theta=1$			
	E	CE	CC	E	CE	CCE	CCC
d=1	•	•	•	•	•	•	•
d=2		•	•		•	•	•
d=3						•	•
H	$\frac{1-q}{2}$	$\frac{q(1-q)}{2}$	$\frac{q^2}{2}$	$\frac{1-q}{2}$	$\frac{q(1-q)}{2}$	$\frac{q^2(1-q)}{2}$	$\frac{q^3}{2}$
T	$\frac{1-q}{2C}$	$\frac{q(1-q)}{C}$	$\frac{q^2}{C}$	$\frac{1-q}{2C}$	$\frac{q(1-q)}{C}$	$\frac{3q^2(1-q)}{2C}$	$\frac{3q^3}{2C}$

Eqm σ: Cont. w. $q \in (0,1)$.

Halfer (H): $\pi = \rho(\theta)\,\Pr_\sigma(\mathbf{a})$.

Thirder (T): $\pi \propto \rho(\theta)\,\Pr_\sigma(\mathbf{a})|D(\theta,\mathbf{a})|$.

Normalising constant: $C = 1 + q + \frac{q^2}{2}$.

Planning-Optimal and Interim-Optimal

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Newcomb Tension

Γ exhibits a *Newcomb tension* if $\sigma^* \neq \hat{\sigma}$.
The *value of commitment* is $V^{\text{plan}}(\sigma^*) - V^{\text{plan}}(\hat{\sigma}) > 0$.

One-Boxer Representation

One-Boxer Beliefs

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- **Halfer need not work.** In the AMD w. $\sigma = \mathrm{Cont}$, $D = \{X, Y\}$; one-boxer beliefs require $P^{\mathrm{OB}}(X) \geq \frac{3}{4}$, not $\frac{1}{2}$.

SBBtW example

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- **Committed thirder**: plays σ^* because bound.
- **Uncommitted one-boxer**: plays σ^* because beliefs make it interim-optimal.
- In any game with a Newcomb tension, we can model an agent with 'commitment power' as having one-boxer beliefs.
- In some social learning games (e.g. BGYT, SBLTF), this amounts to assuming all agents are halfers.

Single-Agent Results

Randomisation Resolves the Tension

Theorem (Randomisation Resolution)

In any single-agent, single-information-set SLU game, the planning-optimal behavioural strategy σ^* is also interim-optimal. This holds for any $|\Theta| \geq 1$.

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- Generalises Piccione–Rubinstein: not only does $q^* = 2/3$ work in the single-state AMD, but a planning-optimal σ^* *always* coincides with the interim fixed point in single-agent games.
- Two birds with one stone: randomisation creates the fixed point *and* aligns it with the planning optimum.

Intuition / pure-strategy FPs

Multi-Agent Games

Multi-Agent Social Welfare

Let $D(\theta, n, \mathbf{a})$ = dots occupied by agent n in compound state $(\theta, \mathbf{a})$. Under utilitarian SWF:

$$V^{\text{plan}}(\sigma) = \sum_{\theta,\mathbf{a}} \rho(\theta) \Pr_{\sigma}(\mathbf{a}) \sum_{n \in N} \frac{|D(\theta, n, \mathbf{a})|}{|D(\theta, \mathbf{a})|} U_n(\mathbf{a}, \theta).$$

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- Planner's social weight of agent n:

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- Interim agent: same formula but with $\pi(\theta, \mathbf{a})$ (thirder) in place of $\rho(\theta) \Pr_{\sigma}(\mathbf{a})$.

Multi-Agent Newcomb Tension

Theorem

(i) **Symmetric dot structure $\Rightarrow$ no tension.** If $|D(\theta, n, \mathbf{a})|$ is the same across agents n for every $(\theta, \mathbf{a})$, then per-dot continuation reasoning recovers σ^*.

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(i) **Symmetric dot structure $\Rightarrow$ no tension.** If $|D(\theta, n, \mathbf{a})|$ is the same across agents n for every $(\theta, \mathbf{a})$, then per-dot continuation reasoning recovers σ^*.

(ii) **Asymmetric dot structure $\Rightarrow$ generic tension.** If $|D(\theta_0, n_0, \mathbf{a}_0)| \neq |D(\theta_0, n_1, \mathbf{a}_0)|$ for some $n_0, n_1, \theta_0, \mathbf{a}_0$, then generically $\sigma^* \neq \hat{\sigma}$.

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- **Interim (thirder) weights**: $\lambda^{\text{int}}_{\text{orig}} = 2/3$, $\lambda^{\text{int}}_{\text{clone}} = 1/3$.

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Randomisation is powerless

Each agent has at most one payoff-relevant dot. Mixing over reports is weakly dominated; the tension persists.

Duplicating AMD applications

Conclusion

Related Literature

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- **SLU in Philosophy:** Kierland and Monton (2005), Ross (2010), Janda (2024), Spohn (2025), Schwarz (2015).
  - Elga (2000), Lewis (2001), Winkler (2017).

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- **Single-agent:** randomisation always resolves the tension (and creates the fixed point where pure strategies fail).
- **Multi-agent:** the tension requires asymmetric dot structure; otherwise it generically vanishes.
- **Applications:** Duplicating Sleeping Beauty (tension that survives randomisation), Duplicating AMD (Persistent Tension Proposition).

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- This is $C(\sigma^*)$ times the *interim expected advantage* of a'.
- At a planning optimum the interim advantage is non-positive in every direction $\Rightarrow \sigma^*$ is interim-optimal.

Pure-Strategy Fixed Points Are Planning-Optimal

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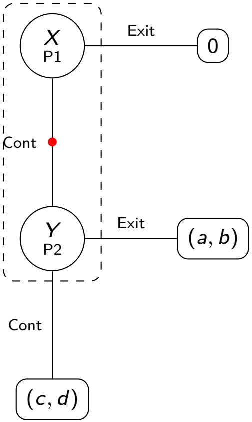
- When a pure-strategy interim fixed point *exists*, it is already planning-optimal.
- Corollary: randomisation only *matters* when no pure fixed point exists (as in AMD). In such cases, it simultaneously delivers existence and planning-optimality.

Back

Duplicating Absent-Minded Driver



- P1 occupies X; if P1 continues, the clone (P2) is created (red dot) and occupies Y.

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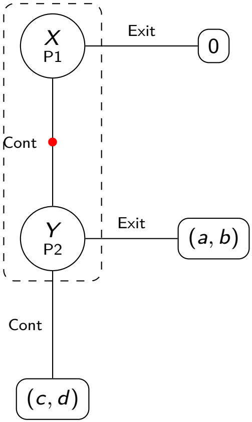

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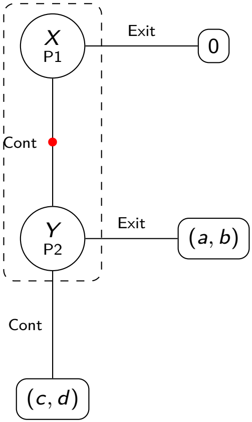

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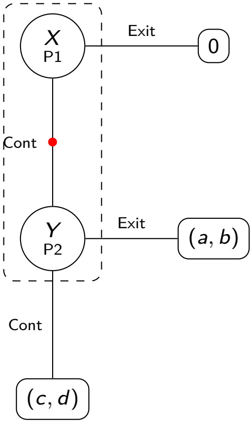

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- The dot structure depends on the action profile.

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- With $u_1 = u_2$, the compound-state welfare just equals the common payoff $\Rightarrow$ reduces to a single-agent optimisation $\Rightarrow$ the single-agent theorem applies.
- **Duplication alone does not create the tension.**

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- Randomisation does **not** help.

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Add a clone to each state of the two-state AMD (clone created at the first Continue):

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- **Equal payoffs $\Rightarrow$ no tension, even with asymmetric dots across agents.**

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- Standard imperfect information — distinct branches, not co-reachable — is **not** SLU.

Planning stage

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- P^{OB} is one-boxer iff $\sum_d P^{\mathrm{OB}}(d)\, g_d \geq 0$, i.e. $P^{\mathrm{OB}}(X_0) + P^{\mathrm{OB}}(X_1) \ \geq \ 3\, P^{\mathrm{OB}}(Y_0) + 2\, P^{\mathrm{OB}}(Z_1) + 4\, P^{\mathrm{OB}}(Y_1)$.

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- Any distribution putting *enough* weight on X_0, X_1 does the job. Putting all weight there is sufficient; scattering weight to the continuation nodes is permitted up to this bound.

Halfer Beliefs

- A halfer updates on the event 'I am at *h*' using the *unconditional* probability of reaching *h* in each state-action profile:

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- Here, the halfer's beliefs are just the unconditional probabilities over $(\theta, \mathbf{a})$.

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The Planning Stage

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Newcomb's Problem as an SLU Game

$A(h_0) = \{\text{Commit}, \text{Not Commit}\}$, $A(h) = \{\text{One-box}, \text{Two-box}\}$,
$\theta \in \{\text{Full}, \text{Empty}\}$ *chosen by the predictor as a function of* h_0.
Then $a^{\text{plan}} = \text{One-box} \neq \text{Two-box} = a^{\text{int}}$.

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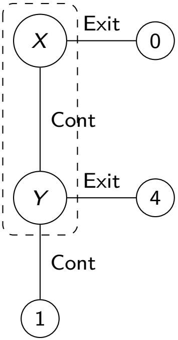
AMD: Setup



Under σ: Cont. w. $q \in (0, 1)$:

Profile	Prob	Dots
E	$1 - q$	$\{X\}$
CE	$q(1 - q)$	$\{X, Y\}$
CC	q^2	$\{X, Y\}$

Per-dot gain from Continue (vs. Exit) with others on σ:

$$g_X = 4 - 3q, \qquad g_Y = -3.$$

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Planning. $V^{\text{plan}}(q) = q(1-q)\cdot 4 + q^2 \cdot 1 = 4q - 3q^2$, so

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Thirder. Compound-state weights $\propto \rho(\theta)\,\mathrm{Pr}_\sigma(\mathbf{a})|D(\theta,\mathbf{a})|$; normalising constant $C(q) = 1 + q$. Uniform within compound states gives

$$P^T(X) = \tfrac{1}{1+q}, \qquad P^T(Y) = \tfrac{q}{1+q}.$$

Interim FOC:

$$\tfrac{4-3q}{1+q} - \tfrac{3q}{1+q} = \tfrac{4-6q}{1+q} = 0 \;\Rightarrow\; \hat{q}_T = \tfrac{2}{3} = \sigma^*.$$

AMD: Halfer

Halfer. Compound-state weights $\propto \rho(\theta)\,\mathrm{Pr}_\sigma(\mathbf{a})$ (no dot-count scaling). Uniform within compound states:

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Interim FOC:

$$(1 - \tfrac{q}{2})(4 - 3q) - \tfrac{3q}{2} = 0$$

$$\Rightarrow\ 3q^2 - 13q + 8 = 0\ \Rightarrow\ \hat{q}_H = \tfrac{13-\sqrt{73}}{6} \approx 0.743.$$

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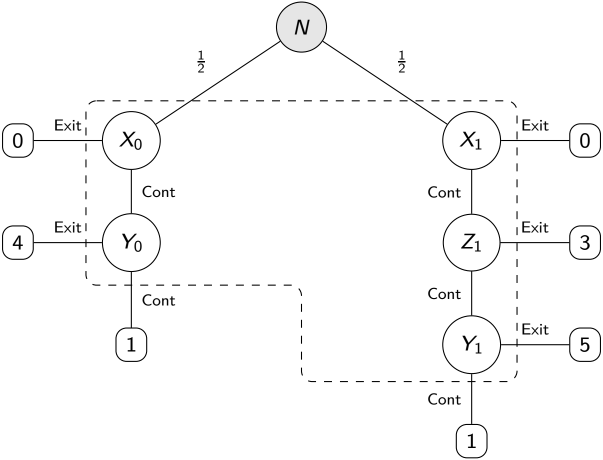

- $\theta = 0$: standard AMD. $\theta = 1$: an extra intersection Z_1 inflates the dot count.

SB Behind the Wheel: Planning = Interim

State payoffs under σ:

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Interim first-order condition gives the same quadratic:

$$12\sigma^2 + 2\sigma - 7 = 0 \;\Rightarrow\; \hat{\sigma} = \sigma^*.$$