Sleeping Newcomb

The 'Newcomb Tension' in Games with Self-Locating Uncertainty

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Learning with Temporal Ignorance

- I am broadly interested in Bayesian learning, and modelling this in the presence of 'temporal ignorance'.
  - Agents arrive in a game at time t and observe some signal whose distribution depends on this arrival time, but do not perfectly observe it.
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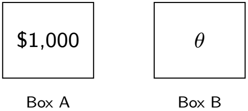
- This temporal ignorance is an instance of *self-locating uncertainty* (SLU): uncertainty as to which node one occupies *within* a given information set and play of a game.
- I study a dynamic inconsistency in such games that I call a *Newcomb tension*, and when it can (and cannot) be resolved.

Three Puzzles



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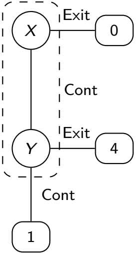
I. Newcomb



II. AMD

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Different paradoxes, same structure: locally rational reasoning at the point of decision disagrees with the planning optimum. I call this a *Newcomb tension*, and I will organise the talk around these three puzzles.

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2. **Single agent:** randomisation always resolves the tension (this is not a coincidental property of the AMD).
3. **Multiple agents:** the tension can survive randomisation, and where it does, even the belief representation can fail: commitment becomes irreplaceable. The planner in Duplicating Sleeping Beauty bets like a halfer even though the correct credences are thirder.

The Model

- **Game:** single information set h; agents N; prior ρ on Θ; for each $(\theta, \mathbf{a})$, $D(\theta, \mathbf{a})$ = the *dots* (nodes of h) visited. Behavioural $\sigma \in \Delta(A(h))$: independent randomisation at each dot.

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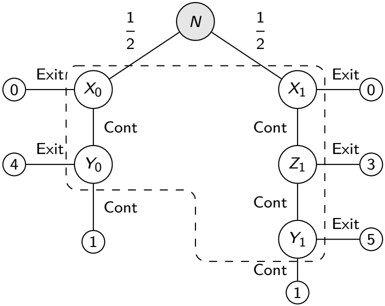
Newcomb tension

Γ exhibits a *Newcomb tension* if $\sigma^* \neq \hat{\sigma}$.

Halfer vs Thirder: Sleeping Beauty Behind the Wheel



	$\theta = 0$			$\theta = 1$			
	E	CE	CC	E	CE	CCE	CCC
d=1	•	•	•	•	•	•	•
d=2		•	•		•	•	•
d=3						•	•
H	$\frac{1-q}{2}$	$\frac{q(1-q)}{2}$	$\frac{q^2}{2}$	$\frac{1-q}{2}$	$\frac{q(1-q)}{2}$	$\frac{q^2(1-q)}{2}$	$\frac{q^3}{2}$
T	$\frac{1-q}{2C}$	$\frac{q(1-q)}{C}$	$\frac{q^2}{C}$	$\frac{1-q}{2C}$	$\frac{q(1-q)}{C}$	$\frac{3q^2(1-q)}{2C}$	$\frac{3q^3}{2C}$

Eqm σ: Cont. w. $q \in (0,1)$.

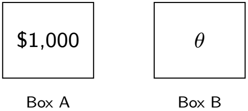
Halfer (H): $\pi = \rho(\theta)\,\Pr_\sigma(\mathbf{a})$.

Thirder (T): $\pi \propto \rho(\theta)\,\Pr_\sigma(\mathbf{a})|D(\theta,\mathbf{a})|$.

Normalising constant: $C = 1 + q + \frac{q^2}{2}$.

I. Newcomb's Problem

Three Puzzles



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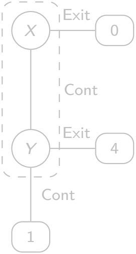

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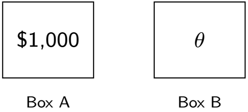

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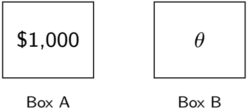

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- **Newcomb's game**: add a planning stage h_0 before nature.

  $$h_0 \longrightarrow \text{predictor sets } \theta(h_0) \longrightarrow h$$

  Commit-to-One-box $\Rightarrow \theta =$ Full, payoff M; otherwise $\theta =$ Empty.

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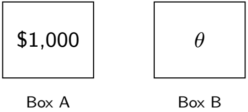

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Newcomb tension (informal)

The planning-optimal and interim-optimal actions diverge: $a^{\text{plan}} =$ One-box, but at h Two-box dominates, so $a^{\text{int}} =$ Two-box. The *value of commitment* is $M - 1 > 0$.

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- **Non-uniqueness (dot-independent payoffs).** If $v_d(a', \mathbf{a}_{-d}, \theta) = v(a', \theta)$, then $P^{\mathrm{OB}}(\theta) = \rho(\theta)$ with *any* conditional over $(\mathbf{a}, d)$ suffices; in particular halfer beliefs do the job.

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- **Halfer need not work.** In the AMD with $\sigma = \mathrm{Cont}$, $D = \{X, Y\}$; one-boxer beliefs require $P^{\mathrm{OB}}(X) \geq \frac{3}{4}$, not $\frac{1}{2}$.

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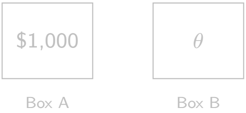
- So commitment can usually be modelled as a belief, with no explicit commitment technology.

II. The Absent-Minded Driver: One Agent

Three Puzzles



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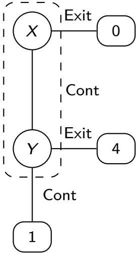

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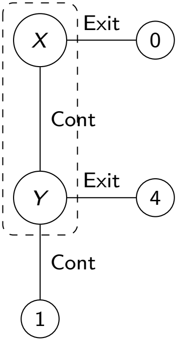
III. Sleeping Beauty

Puzzle II: one agent, self-locating within a single play. Does allowing randomisation resolve the tension?

The Absent-Minded Driver (Piccione and Rubinstein, 1997)



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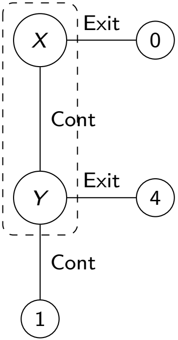

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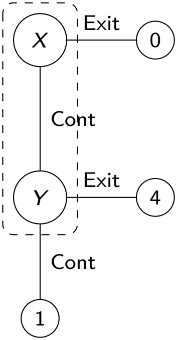

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- Each pure strategy makes the *other* interim-optimal: a Newcomb tension, and no pure fixed point at all.

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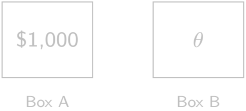
- Two birds with one stone: randomisation *creates* the fixed point (none exists in pure strategies) and *aligns* it with the planning optimum.

Proof idea

III. Duplicating Sleeping Beauty: Many Agents

Three Puzzles



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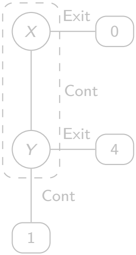

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Puzzle III: several agents sharing an information set. Here the tension can survive randomisation, and even belief representation can fail.

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- Coin toss $\theta \in \{H, T\}$. Original wakes Monday in both states; *if Tails, a clone is created and wakes Tuesday*. Awakenings indistinguishable; each reports a credence $x \in [0, 1]$ in Tails, scored by quadratic loss.

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- Assuming co-cardinal utilities, here the tension results from a shift in the social weights assigned to the planning and interim stages.

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III. Sleeping Beauty

Duplicating Sleeping Beauty

- Awakenings per agent:

	$\theta = H$	$\theta = T$
Original	1	1
Clone	0	1

Duplicating Sleeping Beauty

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- **Planner** (compound states, uniform over dots): weights $\frac{1}{2}, \frac{1}{4}, \frac{1}{4}$ on the three awakenings; by agent, $\lambda_{\text{orig}} = \frac{3}{4}$, $\lambda_{\text{clone}} = \frac{1}{4}$.
- **Awakened thirder**: $\frac{1}{3}$ each; $\lambda^{\text{int}}_{\text{orig}} = \frac{2}{3}$, $\lambda^{\text{int}}_{\text{clone}} = \frac{1}{3}$.

The Tension: Halfer Bets, Thirder Credences

Proposition (Newcomb Tension in Duplicating Sleeping Beauty)

With quadratic loss $S(x, \theta) = -(x - \mathbf{1}\{\theta = T\})^2$, the planning-optimal report is the **halfer credence** and the interim-optimal report the **thirder credence**:

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- Thirder credences are (I maintain) *correct*; but the right *bet* ex ante is the halfer's. Kierland and Monton (2005)'s halfer recommendation for this problem is exactly my planning stage; the disagreement is confined to credence.

When Does the Multi-Agent Tension Arise?

Theorem (Multi-Agent Newcomb Tension)

(i) **No tension** if the game is *payoff-independent* (each agent's payoff depends only on actions at their own dots) *and* dot counts are symmetric across agents.

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(ii) **Generic tension** if payoffs are *interdependent*, or dot counts are asymmetric across agents.

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- Dup SB: payoff-independent but count-asymmetric. Duplicating AMD: both.

Duplicating AMD

Conclusion

Related Literature

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- **Dynamic consistency:** Strotz (1955), Machina (1989); closest apparatus: Vergopoulos (2011)'s two-state-space framework.

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- **Single agent:** randomisation always resolves the tension (and creates the fixed point where pure strategies have none).
- **Multiple agents:** interdependence or asymmetric awakenings revive the tension; it survives randomisation, and can even defeat every belief. Commitment becomes irreplaceable.

Proof Idea

Sketch of the single-agent argument:

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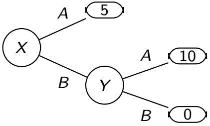
- At a planning optimum the interim advantage is non-positive in every direction, so σ^* is interim-optimal.

Back

Non-existence of One-Boxer Beliefs

Two single-agent, pure-strategy failures (behavioural strategies rescue both):



Pure optimum A truncates Y: δ_B dominates at the only reached dot; no beliefs on reached dots.

	$a_2\!=\!0$	$a_2\!=\!1$	$a_2\!=\!2$
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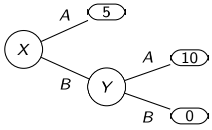

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- The criterion (theorem of the alternative): beliefs exist iff no dot-wise dominating mixture on reached dots. Behavioural planning optima escape both traps for a single agent; interdependence does not.

◂ Back

AMD: the Halfer's Fixed Point

Thirder. $P^T(X) = \dfrac{1}{1+q}$, $P^T(Y) = \dfrac{q}{1+q}$; interim FOC

$$\frac{4-6q}{1+q} = 0 \;\Rightarrow\; \hat{q}_T = \frac{2}{3} = \sigma^*.$$

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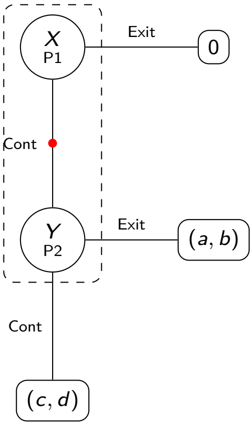
- The halfer's fixed point is *not* the planning optimum: halfer beliefs are not innocuous in dot-dependent games.

Back

Duplicating Absent-Minded Driver



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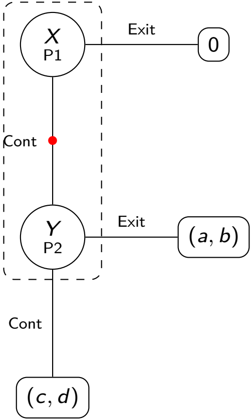

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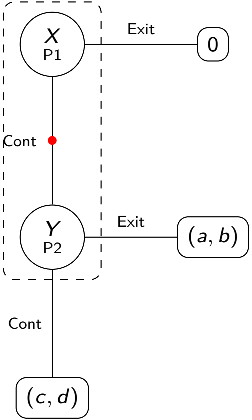

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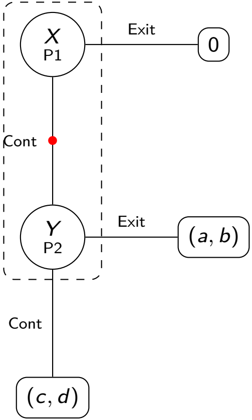

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- The dot structure depends on the action profile, and payoffs are *interdependent*: P1's payoff depends on the action at P2's dot.

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- With $u_1 = u_2$, compound-state welfare equals the common payoff: the problem reduces to a single-agent optimisation and the single-agent theorem applies.
- **Duplication alone does not create the tension.**

Duplicating Sleeping Beauty Behind the Wheel

Add a clone to each state of the two-state AMD (created at the first Continue):

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- **Equal payoffs $\Rightarrow$ no tension, even with asymmetric dots across agents.**

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